A = \{ 2, 3, 5, 8, 11, 13 \}  B = \{ 2, 3, 5, 7, 11 \}

Union \ A \cup B = \{ x : x \in A \text{ or } x \in B \} \ A \cup B = \{ 2, 3, 5, 7, 8, 11, 13 \}

Intersection \ A \cap B = \{ x : x \in A \text{ and } x \in B \} \ A \cap B = \{ 2, 3, 5, 11 \}

Disjoint sets have empty (pair-wise) intersection.

Relative complement \ A \setminus B = \{ x \in A : x \not\in B \} \ A \setminus B = \{ 8, 13 \} \ B \setminus A = \{ 7 \}

Symmetric difference \ A \oplus B = \{ x : x \in A \text{ or } x \in B, \text{ but } x \text{ is not in both} \} \ A \oplus B = \{ 7, 8, 13 \}

Universe or universal set – identifies the set of interest.

If \ A \subset U, then U \setminus A \text{ is Absolute complement} \text{ or complement } A^C = \{ x \in U : x \not\in A \}

Venn diagrams

If Set A = Regions 1 and 2, and Set B = Regions 2 and 3:

Union \ A \cup B = \text{Regions 1, 2, 3}

Intersection \ A \cap B = \text{Region 2}

Relative complement \ A \setminus B = \text{Region 1}

Symmetric difference \ A \oplus B = \text{Regions 1, 3}

If the Universal set = Regions 1, 2, 3, 4 then Absolute complement \ A^C = \text{Regions 3, 4}

Table 1. Laws of Algebra of sets [For \cup \text{ and } \cap] \text{: (p25)}.

Note well: “... proving set-theoretic identities is not a primary focus in discrete mathematics, so the proofs [in Examples 3 and 4 on page 24] are just here for completeness.”

Additional note: Symmetric difference \oplus \text{ is associative [proof p26].} \text{ Is it commutative?}

An \text{ ordered pair} \ (s, t) \text{ has a first element } s \in S \text{ and a second element } t \in T

The set of all such ordered pairs is the \text{ product set} \ S \times T = \{ (s, t) : s \in S \text{ and } t \in T \}

S^2 \text{ is } S \times S. \text{ Also the product set of n sets consists of ordered n-tuples.}
1. U=\{1,2,3,...,12\}, A=\{1,3,5,7,9,11\}, B=\{2,3,5,7,11\}, C=\{2,3,6,12\}, D=\{2,4,8\}
   a) A \cap B = \{1,2,3,5,7,9,11\}
   b) A \cap C = \{3\}
   c) (A \cap B) \cap C = \{3\}
   d) A \cap B = \{1,9\}
   e) C \cap D = \{3,6,12\}
   f) B \cap D = \{3,4,5,7,8,11\}
   g) \vert C \vert = 4 \rightarrow \vert \rho(C) \vert = 2^4 = 16

2. A=\{1,2,3\}, B=\{n \in \mathbb{P} : n \text{ is even}\}, C=\{n \in \mathbb{P} : n \text{ is odd}\}
   a) A \cap B = \{2\}
   b) A \cap C = \{\}
   c) A \cap C = \{\}
   d) B \cap C = \{3,4,5,7,8,11\}
   e) |C| = 4
   f) |Y \setminus V(C)| = 2

3. U=\mathbb{Reals}, [a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}
   a) \[0,3\] \cap [2,6] = [2,3]
   b) \[0,3\] \cup [2,6] = [0,6]
   c) \[0,3\] \cap [2,6] = [0,2]
   d) \[0,3\] \cup [2,6] = [0,2] \cup (3,6]
   e) \[0,3\] \cap \emptyset = \emptyset
   f) \[0,\infty\] \cap \emptyset = \emptyset
   g) \[0,\infty\] \cap \mathbb{Z} = \{0,1,2,3,...\} = \mathbb{N}
   h) \[0,\infty\] \cap (-\infty,2] = [0,2]
   i) ((\[0,\infty\] \cup (-\infty,2)) \cap \mathbb{C} = (-\infty,\infty) \cap \mathbb{C} = \emptyset

4. \Sigma=\{a,b\}, A=\{a, b, aa, bb, aaa, bbb\}, B=\{w \in \Sigma^* : \text{length}(w) \geq 2\}, C=\{w \in \Sigma^* : \text{length}(w) \leq 2\}
   a) A \cap C = \{a, b, aa, bb\}
   b) A \cap C = \{a, b, aa, bb\}
   c) A \cap C = \{a, b, aa, bb\} = (A \cap C) \cup (C \setminus A)
   d) B \cap C = \{\}
   e) B \cap C = \{w \in \Sigma^* : \text{length}(w) = 2\} = \{aa, ab, ba, bb\}
   f) B \cap C = \{w \in \Sigma^* : \text{length}(w) < 2\} = \{a, b, aa, ab, ba, bb, aaa, aab,aba, bba, aaaa, ...\}
   g) \Sigma \setminus B = \{a, b\}, \Sigma \setminus C = \emptyset
   h) \Sigma \setminus C = \emptyset
   i) \rho(\Sigma) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}, \text{a set of partial alphabets}

5. \Sigma=\{a,b\}, U=\Sigma^*, A=\{a, b, aa, bb, aaa, bbb\}, B=\{w \in \Sigma^* : \text{length}(w) \geq 2\}, C=\{w \in \Sigma^* : \text{length}(w) \leq 2\}
   a) B \cap C = \{w \in \Sigma^* : \text{length}(w) < 2 \text{ and } \text{length}(w) > 2\} = \emptyset
   b) (B \cap C) \cap = \{\lambda, a, b, aa, aab, ab, abb, ...\}
   c) (B \cap C) \cap = \{\lambda, a, b, aa, aab, ab, abb, ...\}
   d) (B \cap C) \cap = \emptyset
   e) A \cap C = \{\lambda, a, b, ab\} = C \setminus A
   f) A \cap C = \{w \in \Sigma^* : \text{length}(w) < 2 \text{ and } w \notin A\} = \{\lambda\}
   g) B \cap C = \{B \cup C\} \cap \text{ and } B \cap C = \emptyset \text{ by DeMorgan laws.}

6. Give counterexamples to each of the following false statements:
   a) For any A not = B, it is false that A \cup B \subseteq A \cap B.
   b) For any nonempty A, it is false that A \cap \emptyset = A
   c) For any C that is not a subset of A, it is false that A \cap (B \cup C) = (A \cap B) \cup C.
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7. For any set $A$, $A \oplus A = \emptyset$ and $A \oplus \emptyset = A$.

8. If $A = \{1, 3, 5, 7, 9, 11\}$ and $B = \{2, 3, 5, 7, 11\}$, then
   a) $|A| = 6$  b) $|B| = 5$
   c) $|A \cup B| = |\{1, 2, 3, 5, 7, 9, 11\}| = 7$  d) $|A| + |B| - |A \cap B| = 6 + 5 - |\{3, 5, 7, 11\}| = 11 - 4 = 7$
   e) $|A \cup B|$ must count elements in $A \cap B$ only once not twice.

9. Give counterexamples to each of the following false statements:
   a) If $A = \emptyset$ then $A \cap B = A \cap C$ does not imply $B = C$.
   b) If $A = B \cap C$ then $A \cap B = A \cap C$ does not imply $B = C$.
   c) If $B \cap C = \emptyset$ and $A = B \cap C$ for nonempty sets $B$ and $C$, then $A \subseteq B \cup C$ does not imply $A \subseteq B$ or $A \subseteq C$.

10. Relative complement is neither commutative nor associative.
    Say $A = \{1, 2, 3, 6\}$   $B = \{1, 3, 4, 7\}$   $C = \{1, 2, 4, 5\}$
    a) then $A \setminus B = \{2, 6\}$ and $B \setminus A = \{4, 7\}$ and $A \setminus B \neq B \setminus A$.
    b) also $A \setminus (B \cap C) = A \setminus \{3, 7\} = \{1, 2, 6\}$ and $(A \setminus B) \setminus C = \{2, 6\} \setminus C = \{6\}$ and $A \setminus (B \cap C)$ is not $= (A \setminus B) \setminus C$.

11. Let $A = \{a, b, c\}$ and $B = \{a, b, d\}$ then
    a) $A \times A = \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$
    b) $A \times B = \{(a, a), (a, b), (a, d), (b, a), (b, b), (b, d), (c, a), (c, b), (c, d)\}$
    c) $\{(x, y) \in A \times B : x = y\} = \{(a, a), (b, b)\}$

12. Let $S = \{0, 1, 2, 3, 4\}$ and $T = \{0, 2, 4\}$, then
    a) $|S \times T| = 3 \cdot 5 = 15$   $|T \times S| = 3 \cdot 5 = 15$
    b) $\{(m, n) \in S \times T : m < n\} = \{(0, 2), (0, 4), (1, 2), (1, 4), (2, 4), (3, 4)\}$
    c) $\{(m, n) \in T \times S : m < n\} = \{(0, 1), (0, 2), (0, 3), (0, 4), (2, 3), (2, 4)\}$
    d) $\{(m, n) \in S \times T : m + n \geq 3\} = \{(0, 4), (1, 2), (1, 4), (2, 2), (2, 4), (3, 0), (3, 2), (3, 4), (4, 0), (4, 2), (4, 4)\}$
    e) $\{(m, n) \in T \times S : mn \geq 4\} = \{(2, 2), (2, 3), (2, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$
    f) $\{(m, n) \in S \times S : m + n = 10\} = \emptyset$ because the largest possible sum is 8.

13. For each of the following sets, list all elements if it has fewer than seven; else list seven elements.
    a) $\{(m, n) \in \mathbb{N}^2 : m = n\} = \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), \ldots\}$
    b) $\{(m, n) \in \mathbb{N}^2 : m + n \text{ is prime}\} = \{(0, 2), (0, 3), (1, 2), (2, 1), (3, 0), (0, 5), (1, 4), \ldots\}$
    c) $\{(m, n) \in \mathbb{P}^2 : m = 6\} = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (6, 7), \ldots\}$
    d) $\{(m, n) \in \mathbb{P}^2 : \min \{m, n\} = 3\} = \{(3, 3), (3, 4), (3, 5), \ldots, (4, 3), (5, 3), (6, 3), (7, 3), \ldots\}$
    e) $\{(m, n) \in \mathbb{P}^2 : \max \{m, n\} = 3\} = \{(1, 3), (2, 3), (3, 3), (3, 2), (3, 1)\}$
    f) $\{(m, n) \in \mathbb{P}^2 : m^2 = n\} = \{(0, 0), (1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36), \ldots\}$