1. Outline briefly the process used to derive the formula for computing the number of multiples of \( k \) between integers \( m \) and \( n \), where \( m \leq n \) and \( k \) is a positive integer. Various problem-solving methods were used, including: removing ambiguity, solving special cases first, counting a larger set and then pruning.

2. Find all of the factors of 360.

\[
\{ 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360 \}
\]

3. List all the elements in the set \( \{ n \in \mathbb{Z} : n^2 \leq 9 \} \).

\(-3, -2, -1, 0, 1, 2, 3\)

4. If \( \Sigma = \{a, b\} \), list all the elements in the set \( \{ w \in \Sigma^* : \text{length}(w) \leq 3 \} \).

\(\{\lambda, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb\}\)

5. Determine the set \([0,3] \setminus [2,4]\).

\( [0, 2) = \{ x \in \mathbb{R} : 0 \leq x \text{ and } x < 2 \} \)

6. For any sets \( A \) and \( B \), prove that \( (A^c \cap B^c) = (A \cup B)^c \).

\[\begin{array}{c|c|c|c}
A & ii & iii & iv \\
B & i & iii & iv \\
other & iv & iv & iv \\
\end{array}\]

7. For \( n \in \mathbb{P} \), let \( M_n = \{ k \in \mathbb{P} : k \text{ is a multiple of } n \} \). Determine \( M_4 \cap M_6 \).

\( M_4 = \{4, 8, 12, 16, 20, 24, 28, ...\} \)

\( M_6 = \{6, 12, 18, 24, 30, ...\} \)

\( M_4 \cap M_6 = \{12, 24, 36, ...\} = M_{12} \)

8. For \( S = \{1, 2, 3, 4\} \) and \( T = \{1, 2\} \), sketch the set \( \{(m, n) \in S \times T : m + 2n = 5\} \).

\( \{(1, 2), (3, 1)\} \)

\[\begin{array}{cccc|cccc}
1 & 2 & \bullet & 0 & 0 & 0 & 0 \\
\hline
1 & 2 & 3 & 4 \\
\end{array}\]

9. If \( f(n) \) is the sequence function defined by \( f(n) = n + \frac{1}{n} \) for \( n \geq 1 \), then write out the first five terms of the sequence; and show that only finitely many terms will exceed \((1.1)(n)\).

\( f(1) = 2, f(2) = 2.5, f(3) = 10/3, f(4) = 4.25, f(5) = 5.2 \)

\( (n + 1/n) > 1.1n \text{ iff } (1/n) > .1n \text{ iff } 10 > n^2 \text{ iff } n < \sqrt{10} \).

10. If \( S = \{1, 2, 3, 4\} \) and \( T = \{x, y, z\} \), list a function that maps \( S \) onto \( T \). Is there a one-to-one function that maps \( S \) onto \( T \)? Explain your answer.

Answers vary; image of \( S \) under function must be \( T \). \( \{(1, x), (2, y), (3, z), (4, x)\} \)

There is no one-to-one function because \( |T| < |S| \)

11. The contrapositive of “It is raining only if there are clouds.” [If \( r \) then \( c \)] is “If there are not clouds, then it is not raining.” [If \( \neg c \) then \( \neg r \]}

\( \text{MAT251 Review of Unit one} \quad \text{Brief solutions} \)