CN4-6 Representations of Relations

A relation on a set S may be represented in four related ways which may be used to determine its properties: listing of its pairs, a picture of its mappings, a Cartesian graph of its elements, and a matrix showing its correspondences. Each method has advantages and disadvantages.

EXAMPLE The relation $GT$ on $\{1, 2, 3\}$ defined by $a GT b$ if and only if $(a - b)$ is positive:

Its listing is $\{(2,1), (3,1), (3,2)\}$

Its picture might be drawn where the arrows point from $a$ to $b$ in each pair.

Its Cartesian graph might be where only the dark dots are part of the graph

Its matrix would be where each 1 represents a corresponding pair.

These representations may be examined to determine properties of a relation. The listing of pairs can easily show reflexivity and symmetry, but may be tedious for checking transitivity. The picture easily shows reflexivity, symmetry, and (for small relations) transitivity. The Cartesian graph and the matrix representations are seen as congruent by a quarter-turn rotation and easily show the same information on reflexivity and symmetry; moreover, the matrix can be manipulated numerically (squared) to easily test for transitivity.

EXAMPLE The relation on $\{0, 1, 2, 3\}$ defined by the listing $\{(0,1), (0,2), (1,2), (1,3), (2,2), (3,1)\}$ is not Reflexive because it lacks $(0,0)$ and not Irreflexive because it has $(2,2)$; is not Symmetric because it has $(0,1)$ but lacks $(1,0)$ and not Antisymmetric because it has $(1,3)$ and $(3,1)$; is not Transitive because it has $(0,1)$ and $(1,3)$ but not $(0,3)$. The picture might be drawn as Loops at the vertices (nodes) are examined for reflexivity. Directions of arrows are examined for symmetry. Shortcuts for paths of length two are examined for transitivity.

The matrix is the most convenient for examining the properties of large relations. is checked for reflexivity by examining the diagonal; is checked for symmetry by examining the off-diagonal elements; reveals transitivity by squaring the matrix, which corresponds to the composition of the relation with itself.

The existence of a non-zero element in the square, as at $(0,3)$ $(1,1)$ $(2,2)$ and $(2,3)$, where the original matrix has only a zero element shows the relation is not transitive.