In discrete mathematics, graphs are abstract structures that are useful for representing relations. The picture of a graph has a web-like appearance.

Graphs have many uses in computer science and applications, including circuit design, network analysis, schedule of related tasks, route planning.

Genealogy (family trees), computer game-playing, object-oriented design, compiler design are some uses of graph structures.

**Simple graphs** correspond to *irreflexive, symmetric* relations. The structure \( G = (V, E) \) where \( V \) is a set of vertices, and \( E \) is a set of edges *labeled by the endpoints* which are the distinct vertices which represent pairs in the relation. An example would be the *counties* in a state [or region] along with their *common boundaries*.

**Multigraphs** are structures that *may* have multiple edges linking a pair of vertices. \( MG = (V, E, f) \) where \( f \) is a *labeling function* that assigns a *unique* name to each edge. An example would be the major *cities* in a region along with the *highways that connect them*.

**Pseudographs** are generalizations of multigraphs that *may* have edges that connect a vertex to itself. They correspond to *symmetric* relations that *may not be irreflexive*. \( PG = (V, E, f) \) where the *labeling function* may assign a name to a *loop* from vertex \( u \) to itself. An example is campsites in a park along with hiking trails that connect them.

**Directed graphs** correspond to arbitrary relations that *need not be symmetric*. \( DG = (V, E) \) where \( V \) is a set of vertices and \( E \) is a binary relation on \( V \). The edges are *ordered* pairs, not just a pair of vertices. An example is the streets of a city, in which some streets are one-way; the intersections are the vertices. How would a “dead-end” street be represented?

**Directed multigraphs** are structures that *may* have multiple edges from one vertex to another. \( DMG = (V, E, f) \) where the *labeling function* assigns a *unique* name to each ordered pair of vertices. The World Wide Web is a directed multigraph. What are the vertices and what are the edges?

**Note that other names may be used to represent these structures.**