1. Find the first five terms of the sequence defined by each of these recurrences and initial conditions. 
   a) \(a_n = 6a_{n-1}\), \(a_0 = 2\) 
   \[a_0 = 6a_0 = 6 \cdot 2 = 12, \quad a_1 = 6a_1 = 6 \cdot 12 = 72, \quad a_2 = 6a_2 = 6 \cdot 72 = 432, \quad a_3 = 6a_3 = 6 \cdot 432 = 2592\] 

3. Let \(a_n = 2^n + 5 \cdot 3^n\) for \(n = 0, 1, 2, \ldots\) 
   a) \(a_0 = 2^0 + 5 \cdot 3^0 = 1 + 5 = 6, \quad a_1 = 2^1 + 5 \cdot 3^1 = 2 + 15 = 17, \quad a_2 = 2^2 + 5 \cdot 3^2 = 4 + 45 = 49, \quad a_3 = 2^3 + 5 \cdot 3^3 = 8 + 135 = 143\] 

7. Show that the sequence \(\{a_n\}\) is a solution of the recurrence relation \(a_n = a_{n-1} + 2a_{n-2} + 2n - 9\) 
   a) If \(a_n = -n+2\), then \(a_{n-1} + 2a_{n-2} + 2n - 9 = \[-(n-1)+2\] + 2[-(n-2)+2] + 2n - 9 = -n + 1 + 2 - 2n + 4 + 4 + 2n - 9 = -n + 2 = a_n\) 

9. Find the solution to the recurrence relation and initial conditions, using an iterative approach. 
   a) \(a_n = 3a_{n-1}\), \(a_0 = 2\) 
   \[a_1 = 3a_0 = 3 \cdot 2, \quad a_2 = 3a_1 = 3 \cdot 3 \cdot 2 = 3^2 \cdot 2, \quad a_3 = 3a_2 = 3 \cdot 3^2 \cdot 2 = 3^3 \cdot 2, \quad \text{so } a_n = 3^n \cdot 2\] 

11. Suppose the number of bacteria in a colony triples every hour. 
   a) \(a_n = 3a_{n-1}\), \(a_0 = 2\) 
   \[a_1 = 3a_0 = 6, \quad a_2 = 3a_1 = 3 \cdot 6 = 18, \quad a_3 = 3a_2 = 3 \cdot 18 = 54, \quad a_4 = 3a_3 = 3 \cdot 54 = 162, \quad \text{so } a_n = 3^n \cdot 2\] 

15. A loan of $5000 at 7% interest is paid of at a rate of $100 a month. Find a recurrence for \(B(k)\) the balance at the end of \(k\) months. 
   \[B(k) = B(k-1) + \text{interest on } B(k-1) - 100 = (1+.07/12)B(k-1) - 100, \quad \text{where } B(0) = 5000.\] 

17. Prove by induction that \(H_n = 2^n - 1\) is the solution to \(H_n = 2H_{n-1} + 1, \quad H_1 = 1.\) Let \(P(n): H_n = 2^n - 1\) 
   Basis: \(H_1 = 2^1 - 1\), so \(P(1)\) is true. 
   Induction: If \(P(k)\) is true, then \(H_k = 2^k - 1, \quad \text{and } H_{k+1} = 2H_k + 1 = 2(2^k - 1) + 1 = 2^{k+1} - 2 + 1 = 2^{k+1} - 1, \quad \text{so } P(k)\) implies \(P(k+1).\) 

23. Find a recurrence relation for the number of bit strings of length \(n\) that contain a pair of consecutive 0s. 
   We see that there are no pairs in strings of length 0 or 1, so \(a_0 = 0\) and \(a_1 = 0\) 
   For \(n \geq 2\), there are \(2^{(n-2)}\) strings of length \((n-2)\) to which we adjoin 00 
   and to the \(a_{n-2}\) strings with a pair of consecutive 0s we adjoin 10 
   and to the \(a_{n-1}\) strings with a pair of consecutive 0s we adjoin 1 
   Clearly all these cases are unique because of different endings, so \(a_n = a_{n-1} + a_{n-2} + 2^{(n-2)}\) 
   \[a_2 = 0 + 0 + 2^0 = 1, \quad a_3 = 1 + 0 + 2^1 = 3, \quad a_4 = 3 + 1 + 2^2 = 8, \quad a_5 = 8 + 3 + 2^3 = 19, \quad a_6 = 19 + 8 + 2^4 = 43, \quad a_7 = 43 + 19 + 2^5 = 94\] 

41. How many bit sequences of length 7 contain an even number of 0s? 
   We see the empty string (of length 0) has zero (an even number) of 0s, so \(a_0 = 1.\) 
   We see the string 1 is the only string of length 1 that has zero 0s, so \(a_1 = 1\) 
   For \(n \geq 2\), we adjoin 1 to the \(a_{n-1}\) strings with an even number of 0s 
   and we adjoin 0 to the \([2^{(n-1)} - a_{n-1}]\) strings with an odd number of 0s. 
   The total of these cases is \(a_n = 2^{(n-1)}\), for \(n \geq 2.\) Indeed, \(a_n = 2^{(n-1)}, \quad \text{for } n \geq 1.\) 
   Hence \(a_7 = 2^{(7-1)} = 64\) strings of length 7 with an even number of 0s.