3.5 p283.

1. Give a recursive algorithm for computing $n^x$ whenever $n$ is a positive integer and $x$ is an integer. The idea is to add $x$ to a smaller value; we see that $n^x = (n-1)^x + x$ is the recurrence. $1^x = x$ is the base.

PROC mul(n : positive integer, x : integer)
IF n = 1 THEN mul(n, x) := x
ELSE mul(n, x) := mul(n-1, x) + x

5. Give a recursive algorithm for finding the minimum of a finite list of integers. In the following algorithm, we use $\text{min}(a,b) := ((a+b) – \text{abs}(a – b))/2$ as a built-in function.

PROC smallest(a_1, a_2, a_3, . . . , a_n : integer)
IF n = 1 THEN smallest(a_1, a_2, a_3, . . . , a_n) := a_1
ELSE smallest(a_1, a_2, a_3, . . . , a_n) := min(smallest(a_1, a_2, a_3, . . . , a_{n-1}), a_n)

9. Devise a recursive algorithm for finding the greatest common divisor of two nonnegative integers $a$ and $b$ with $a < b$ where $\gcd(a, b) = \gcd(a, b-a) = \gcd(b-a, a) = \gcd(b, a)$
The algorithm calls must have first argument smaller, produce $b$ if $a = 0$, but produce $a$ when $a = b – a$.
[This second stopping condition provides an “early” exit.]

PROC gcd(a, b : nonnegative integers with $a < b$)
IF a = 0 THEN gcd(a, b) := b
ELSE IF a = b−a THEN gcd(a, b) := a
ELSE IF a < b−a THEN gcd(a, b) := gcd(a, b−a)
ELSE gcd(a, b) := gcd(b−a, a)

11. Describe a recursive algorithm for multiplying two nonnegative integers $x$ and $y$ based on the conditionals that $xy = 2(x \cdot \lfloor y/2 \rfloor)$ when $y$ is even and $xy = 2(x \cdot \lceil y/2 \rceil) + x$ when $y$ is odd; $x \cdot y = 0$ if $y = 0$.

PROC product(x, y : nonnegative integers)
IF y = 0 THEN product(x, y) := 0
ELSE IF 0 = y (mod 2) THEN product(x, y) := 2 · product(x, y/2)
ELSE product(x, y) := 2 · product(x, (y−1)/2) + x

13 [NOT 19] Prove that the recursive algorithm devised in Exercise 1 is correct.
Proof by induction: $P(n)$ The algorithm correctly computes $n^x$ for all integers $n > 0$.
BASIS: IF n = 1 THEN mul(n, x) := x = 1 · x
INDUCTION STEP: Given $P(k)$ for $k ≥ 1 : mul(k, x) = k · x$,
compute $mul(k+1, x) := mul(k+1−1, x) + x = mul(k, x) + x = k · x + x = (k+1) · x$ which is $P(k+1)$

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