1. List all the permutations of \{a, b, c\}.

   \text{The 3*2*1 ways to arrange these 3 elements: } abc, acb, bac, bca, cab, cba

3. How many permutations \{a, b, c, d, e, f, g\} end with a?

   \text{Count the permutations of } \{b, c, d, e, f, g\} : 6*5*4*3*2*1 = 720

9. How many ways can 12 horses finish “in the money” if all orders of finish are possible?

   \text{Calculate P(12, 3) = 12*11*10 = 1320}

11. How many bit strings of length ten contain:

   a) exactly four 1s? \text{We choose which four places contain those 1s : C(10, 4) = (10*9*8*7) / (4*3*2*1) = 210}
   b) at most four 1s? \text{We add C(10, 4) + C(10, 3) + C(10, 2) + C(10, 1) + C(10, 0) = 386}
   c) at least four 1s? \text{We subtract from the total number strings those that have only 0, 1, 2 or 3 1s :}
      \[ 2^{10} - [ C(10, 0) + C(10, 1) + C(10, 2) + C(10, 3) ] = 1024 - 1 - 10 - 45 - 120 = 848 \]
   d) an equal number of 0s and 1s? \text{Calculate C(10, 5) = 252}

15. In how many ways can a set of five letters be selected from the English alphabet? \text{C(26, 5) = 65780}

17. How many subsets with more than two elements does a set with 100 elements have?

   \text{We subtract from the total number of subsets those that have 0, 1, or 2 elements :}
   \[ 2^{100} - C(100, 0) - C(100, 1) - C(100, 2) = 2^{100} - 1 - 100 - 4950 \]

19. A coin is flipped ten times where each flip comes up either H or T. How many outcomes

   a) are there in total? \text{Since there are ten flips, each with 2 results, there are } 2^{10} = 1024 \text{ outcomes}
   b) contain exactly 2 Hs? \text{Choose which 2 flips have the H : C(10, 2) = 45 outcomes.}
   c) contain at most 3 Ts? \text{Count 0, 1, 2, or 3 T’s : C(10, 0) + C(10, 1) + C(10, 2) + C(10, 3) = 176}
   d) contain the same number of Hs and Ts? \text{Calculate C(10, 5) = 252}

21. How many permutations of the letters ABCDEFG contain

   a) the string BCD? \text{Count the permutations of ASEFG where $ represents BCD : 5*4*3*2*1 = 120}
   b) the string CFGA? \text{Count the permutations of B&DE where $ represents CFGA : 4*3*2*1 = 24}
   c) the strings BA and GF? \text{Count the permutations of SCDE& where $ is BA and & is GF : 5! = 120}
   d) the strings ABC and DE? \text{Count permutations of $&FG where $ is ABC and & is DE : 4! = 24}
   e) the strings ABC and CDE? \text{Must contain ABCDE; count permutations of SFG : 3! = 6}
   f) the strings CBA and BED? \text{It is not possible for B to be in the specified positions.}

23. How many ways can 8 men and 5 women be placed in line if no two women can be adjacent?

   \text{The 8 men can be positioned in 8! ways and determine 9 positions that may be filled by either no or one woman in P(9, 5) = 15120. The total is 40320*15120 = 609638400 ways.}

27. A club has 25 members

   a) How many ways are there to choose four members to serve on an executive committee? \text{C(25, 4) = 12650}
   b) How many ways are there to choose four officers \{P, Vp, S, T\}? \text{P(25, 4) = 303600}
31. The English alphabet has 21 consonants and 5 vowels. How many strings of six lowercase letters have
a) exactly one vowel? Choose the vowel in 5 ways; choose its position in 6 ways; choose the consonants in
the 5 other positions in $21^5$ ways. $5 \times 6 \times 21^5 = 122523030$ ways
b) exactly two vowels? Choose the vowels in $5^2$ ways; choose the position in $C(6, 2)$ ways; choose the
consonants in $21^4$ ways. $5^2 \times 15 \times 21^4 = 72930375$ ways.
c) at least one vowel? From all strings of six letters, exclude those that have no vowels: $26^6 - 21^6$.
d) at least two vowels? Subtract the answer in a) from the answer is c).