1. \((x+y)^4 = (x+y)(x+y)(x+y)(x+y)\) which has \(2^4 = 16\) distinct terms because there are 2 choices for each factor: 
\[
= xxxx+xyxx+xyxy+xyyy+yyxx+yyxy+yyxy+yxxy+yxyx+yxxx+xxyy+xyyy+yxxy+yxyy+yyxy+yyyy
\]
Grouping terms according to the number of \(y\) factors:
\[
x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4
\]
Using the Binomial Theorem: \((x+y)^4 = \binom{4}{0}x^4y^0 + \binom{4}{1}x^3y^1 + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}x^0y^4\) as above.

3. \((x+y)^6 = \binom{6}{0}x^6y^0 + \binom{6}{1}x^5y^1 + \binom{6}{2}x^4y^2 + \binom{6}{3}x^3y^3 + \binom{6}{4}x^2y^4 + \binom{6}{5}xy^5 + \binom{6}{6}y^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + \ldots + y^6
\]

5. \((x+y)^{100}\) has 101 terms in the full expansion.

7. The coefficient of \(x^9\) in \((2–x)^{19}\): The term \(\binom{19}{9}2^{10}(-1)^9\) has coefficient -94595072

9. The coefficient of \(x^{101}y^{99}\) in the expansion of \((2x–3y)^{200}\) is \(\binom{200}{99}2^{101}(-3)^{99} \approx -39.05 \times 10^{135}\)

15. Show that \(\binom{n}{k} \leq 2^n\) for all positive integers \(n\) and \(k\) with \(0 \leq k \leq n\). Refer to Corollary 1, or note that for a set of size \(n\), the number of subsets of size \(k\) must be less than the total number of subsets.

17. Show that if \(n\) and \(k\) are integers with \(1 \leq k \leq n\) then \(\binom{n}{k} \leq \frac{n^k}{2^{k-1}}\). The result is trivial if \(k = 1\).

By definition, \[\binom{n}{k} = \frac{n!}{(n-k)!k!} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots2\cdot1} = \binom{n}{k} \frac{n-1}{k} \frac{n-2}{k-1} \cdots \frac{n-k+2}{2} \frac{n-k+1}{k-1} (n-k+1)\]
Each of the first \((k-1)\) factors is less than or equal to \((n/2)\) and the last factor is \(< n\), so the result follows.

21. Prove that if \(n\) and \(k\) are integers with \(1 \leq k \leq n\) then \(k\binom{n}{k} = n\binom{n-1}{k-1}\)

a) Combinatorial argument: For a set of size \(n\), the number of subsets of size \(k\) with a “distinguished” element can be counted either by forming the subset and then choosing one of the \(k\) elements, or by first choosing an element from the entire set and including it along with \((k-1)\) other elements from the \((n-1)\) remaining.

b) Algebraic argument: \[k \binom{n}{k} = k \frac{n!}{(n-k)!k!} = \frac{n(n-1)!}{(n-k)!(k-1)!} = n \binom{n-1}{k-1}\]

31. Show that a nonempty set has the same number of subsets with an odd number of elements as it does subsets with an even number of elements. This result follows immediately from the remark on page 329 following Corollary 2.