1. Linear homogeneous recurrence relations with constant coefficients have the form
   \[ a_n = c_1a_{n-1} + c_2a_{n-2} + \ldots + c_ka_{n-k}, \] 
   for real coefficients \( c_i \), and \( c_k \neq 0 \).
   a) \( a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3} \) is linear homogeneous of degree [order] 3
   b) \( a_n = 2na_{n-1} + a_{n-2} \) is not linear homogeneous because coefficient 2n is not constant
   c) is linear homogeneous of degree 4
   d) is not linear homogeneous because of the term 2.
   e) is not linear homogeneous because exponent 2
   f) is linear homogeneous of degree 2
   g) is not linear homogeneous because of the term \( n \)

3. Solve the recurrence relations; with the given initial conditions.
   a) \( a_n = 2a_{n-1} \), for \( n \geq 1 \); \( a_0 = 3 \)
      Characteristic equation.: \( r = 2; a_n = c \cdot 2^n \) for \( n = 0 \) implies \( c = 3 \). \( \textbf{So } a_n = 3 \cdot 2^n \textbf{ for } n \geq 0. \)
   b) \( a_n = a_{n-1} \), for \( n \geq 1 \); \( a_0 = 2 \)
      Characteristic equation.: \( r = 1; a_n = c \cdot 1^n \) for \( n = 0 \) implies \( c = 2 \). \( \textbf{So } a_n = 2 \cdot 1^n = 2 \textbf{ for } n \geq 0. \)
   c) \( a_n = 5a_{n-1} - 6a_{n-2} \), for \( n \geq 2 \); \( a_0 = 1, a_1 = 0 \)
      Characteristic equation: \( r^2 = 5r - 6 \) has solutions \( r_1 = 2 \) and \( r_2 = 3. \)
      \( a_n = c_1 \cdot 2^n + c_2 \cdot 3^n \) for \( n = 0 \) implies \( c_1 + c_2 = 1 \) and
      \( a_n = c_1 \cdot 2^n + c_2 \cdot 3^n \) for \( n = 1 \) implies \( 2c_1 + 3c_2 = 0 \).
      The solution is \( c_1 = 3 \) and \( c_2 = -2 \) \( \textbf{so } a_n = 3 \cdot 2^n - 2 \cdot 3^n \textbf{ for } n \geq 0. \)
   d) \( a_n = 4a_{n-1} - 4a_{n-2} \), for \( n \geq 2 \); \( a_0 = 6, a_1 = 8 \)
      Characteristic equation: \( r^2 = 4r - 4 \) has solutions \( r_1 = 2, r_2 = 2 \)
      \( a_n = c_1 \cdot 2^n + c_2 \cdot n \cdot 2^n \) for \( n = 0 \) implies \( c_1 = 6 \) and
      \( a_n = c_1 \cdot 2^n + c_2 \cdot n \cdot 2^n \) for \( n = 1 \) implies \( 2c_1 + 2c_2 = 8 \).
      The solution is \( c_1 = 6 \) and \( c_2 = -2 \) \( \textbf{so } a_n = 6 \cdot 2^n - 2n \cdot 3^n \textbf{ for } n \geq 0. \)

7. In how many ways can a \( 2 \times n \) rectangular board be tiled using \( 1 \times 2 \) and \( 2 \times 2 \) pieces?
   \( n = 0: \) Empty rectangle; \( T_0 = 1; n = 1: T_1 = 1; n = 2: T_2 = 1 + 1 + 1 = 3; n = 3: T_3 = 3 + 1 + 1 = 5; \)
   \( n = 4: \) Adjoin a vertical \( 2 \) by \( 1 \) to each of the ones for \( n = 3, \) or
   adjoin a horizontal pair to each of the ones for \( n = 2, \) or
   adjoin a \( 2 \) by \( 2 \) square to each of the ones for \( n = 2; \) that is \( T_4 = 5 + 3 + 3 = 11 \)
   In general for \( n \geq 2, T_n = T_{n-1} + T_{n-2} + T_{n-2}; \) where \( T_0 = T_1 = 1. \)

   The solution of this recurrence is \( T_n = \frac{2}{3}(2)^n + \frac{1}{3}(-1)^n = \frac{2^{n+1} + (-1)^n}{3} \)

13. Find the solution to \( a_n = 7a_{n-2} + 6a_{n-3} \) for \( n \geq 3 \), with \( a_0 = 9, a_1 = 10, \) and \( a_2 = 32. \)
   Characteristic equation \( r^3 = 7r + 6 \) has three solutions \(-2, -1, 3)\)
   Hence \( a_n = c_1 \cdot (-2)^n + c_2 \cdot (-1)^n + c_3 \cdot (3)^n. \)
   Using the initial conditions gives three linear equations with solution \( c_1 = -3, c_2 = 8, c_3 = 4. \)
   So \( a_n = (-3) \cdot (-2)^n + (8) \cdot (-1)^n + (4) \cdot (3)^n. \) for \( n \geq 0. \)