APPENDIX

SCIENTIFIC NOTATION

Scientific notation is a shorthand method for expressing and working with very large or very small numbers. We described this method in Unit 3.2 and discuss it a little further here. We can express any number as a few digits times 10 to a power, or exponent. The power indicates the number of times that 10 is multiplied by itself.

For example, \( 100 = 10 \times 10 = 10^2 \). Similarly, \( 1,000,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6 \). Note that we do not always need to write out \( 10 \times \ldots \). Instead we can simply count the zeros. Thus \( 10,000 \) is 1 followed by four zeros, so it is \( 10^4 \). To express the numbers 1 and 10 in scientific notation: \( 1 = 10^0 \) and \( 10 = 10^1 \).

To write a number like 300, we break it into two parts: \( 3 \times 100 = 3 \times 10^2 \). Similarly, we can write \( 352 = 3.52 \times 100 = 3.52 \times 10^2 \). Any number can be expressed in scientific notation as a value between 1 and 10 multiplied by 10 to a power.

We can also write very small numbers (numbers less than 1) using powers of 10. For example, \( 0.01 = 1/100 = 1 \times 10^{-2} \). We can make this even more concise, however, by writing \( 1 \times 10^{-2} \) as \( 10^{-2} \). Similarly, \( 0.0001 = 10^{-4} \). Note that for numbers less than 1, the power is 1 more than the number of zeros after the decimal point.

We can write a number like 0.00052 as \( 5.2 \times 0.0001 = 5.2 \times 10^{-4} \).

Suppose we want to multiply numbers expressed in powers of 10. The rule is simple: We add the powers. Thus \( 10^3 \times 10^2 = 10^{3+2} = 10^5 \). Similarly, \( 2 \times 10^8 \times 3 \times 10^7 = 2 \times 3 \times 10^{8+7} = 6 \times 10^{15} \). In general, \( 10^a \times 10^b = 10^{a+b} \).

Division works similarly, except that we subtract the exponents. Thus \( 10^5/10^3 = 10^{5-3} = 10^2 \). In general, \( 10^a/10^b = 10^{a-b} \).

The last operations we need to consider are raising a number to a power and taking a root. In raising a power-of-ten number to a power, we multiply the powers. Thus “one thousand to the fourth power” is \( (10^3)^4 = 10^{3 \times 4} = 10^{12} \). Care must be used if we have a number like \( (2 \times 10^4) \). Both the 2 and the \( 10^4 \) are raised to the third power, so the result is \( 2^3 \times (10^4)^3 = 8 \times 10^{4 \times 3} = 8 \times 10^{12} \).

Taking a root is equivalent to raising a number to a fractional power. Thus the square root of a number is the number to the ½ power. The cube root is the number to the ⅓ power, and so forth. For example, \( \sqrt{100} = 10^{1/2} = (10^2)^{1/2} = 10^{2 \times 1/2} = 10^1 = 10 \).

SOLVING DISTANCE, VELOCITY, TIME (d, V, t) PROBLEMS

Many problems in this book (and in science in general) involve the motion of something. In such problems, we often know two of the three quantities distance, velocity, and time \( (d, V, t) \), and we want to know the third. For example, we have something moving at a speed \( V \) and want to know how far it will travel in a time \( t \). Or we know that something travels with a speed \( V \) and want to find out how long it takes for the object to travel a distance \( d \). We can usually solve such problems in our heads if the motion involves automobiles. For example, if it is 160 kilometers to a city and we travel at 80 kilometers per hour, how long does it take to get there? Or how far can we drive in 2 hours if we are traveling at 60 miles per hour? Because we solve such problems routinely, you might find it easier to think of astronomical \( (d, V, t) \) problems in terms of cars.

Regardless of your approach, the method of solution is simple. Begin by making a sketch of what is happening. Draw an arrow to indicate the motion. Label the known quantities and put question marks beside the things you want to find. Then write out the basic relation \( d = V \times t \). If you want to find \( d \) and know \( V \) and \( t \), just multiply them for the answer. If you want to find the time and are given \( V \) and \( d \), solve for \( t \) by dividing both sides by \( V \) to get \( t = d/V \). If you want the velocity, divide \( d \) by \( t \): \( V = d/t \).

In some problems the motion may be in a circle of radius \( r \). In that case the distance traveled will be related to the circumference of the circle, \( 2\pi r \). For such cases you may need to use the expression \( V = 2\pi rt \).

In most problems you will find that it is helpful to write the units of the quantities in the equation. For example, suppose you are asked how long it takes to travel 1500 km at a velocity of 30 kilometers per second. Insert the quantities so that

\[ t \frac{d}{V} = \frac{1500 \text{ km}}{30 \text{ km/sec}} = \frac{1500 \text{ km}}{30 \text{ km}} = 50 \text{ sec}. \]

Note that the units of kilometers cancel out and leave us with units of seconds, as the problem requires.