Goal: Develop a counting formula to count multiples of \( k \) between \( m \) and \( n \)

Immediate task: to count the multiples of seven between 251 and 2008

Note: “between” accepts endpoints; “strictly between” rejects endpoints.

1. How many integers are between 1 and 10? [Recall “between” is inclusive of endpoints!]

2. How many integers are between 1 and \( n \)?

3. How many integers are between 10 and 100? [Use over-count and exclusion]

4. How many integers are between positive integers \( m \) and \( n \) (where \( m \leq n \))? \[ n - (m - 1) \]

5. How many integers are between -10 and 100? [Same formula works!]

If \( m \) and \( n \) are integers, with \( m \leq n \), then there are \( n - m + 1 \) integers \( i \) with \( m \leq i \leq n \).

6. How many even integers are between 1 and 99?

7. How many even integers are between 1 and \( n \)? [Between 1 and \( n \), there are \( s \) even integers: \( n - 2 < 2s \leq n \)]

**Notation:** Floor of any real number \( x \), written \( \lfloor x \rfloor \), is largest integer \( \leq x \)

Remark: Between 1 and \( n \), there are \( \lfloor n/2 \rfloor \) even integers

8. How many even integers are between 9 and 99? [Use over-count and exclusion]

9. How many even integers are between \( m \) and \( n \)?

10. How many multiples of three are between 1 and 251?

   Remark: Between 1 and \( n \), there are \( \lfloor n/k \rfloor \) integers that are multiples of \( k \).

11. How many multiples of three are between 100 and 251?

12. How many multiples of three are between \( m \) and \( n \)?

**Theorem 1:** Let \( m \) and \( n \) be integers with \( m \leq n \), and let \( k \) be a positive integer, then the number of multiples of \( k \) between \( m \) and \( n \) is \( \left\lfloor \frac{n}{k} \right\rfloor - \left\lfloor \frac{m-1}{k} \right\rfloor \) which differs from \( (n - m + 1)/k \) by at most 1.

   [Note carefully the argument on page 4 of the text.]

13. How many multiples of seven are between 251 and 2008?

Definition: A prime is an integer greater than 1 which is not the product of two other integers.

14. How many primes are between 1 and 100? [List of primes to 1000 is on page 6.]

**Prime number theorem:** The fraction of numbers between 1 and (large enough) \( n \) that are prime is approximately \( 1/(\ln n) \).

Summary: What generally useful problem solving methods have we used? See page 6.