Permutations and Combinations

The permutation of a number of objects is the number of different ways they can be placed into position; that is, placed in some order. If only some objects are taken from a larger number of objects, the positions of the chosen objects is also important.

With combinations, the order in which objects were chosen or placed is not important, just which objects were chosen. There is only one way to choose all of the available objects, but there are many ways to select only some objects from a larger number of objects.

In brief, a permutation is an ordered selection, and a combination is an unordered selection.

EXAMPLE Given three pictures, how many ways can they be placed in position on a page?
This is a permutation of three objects because position is important. The first position can be filled with any one of the three pictures; the second position can be filled with either of the remaining two pictures; the third place must be filled with the single remaining picture. There are 3×2×1 = 6 ways. This declining product is called factorial of 3 and is written 3!

EXAMPLE From a collection of ten pictures, how many ways can you arrange four of your favorite pictures?
This is a partial permutation because not all of the pictures are chosen. The first position can be filled with any of the ten pictures, the second position could be filled with any of the remaining nine, the third position filled with any of eight, and the fourth position filled with any of seven. There are 10×9×8×7 = 5040 ways.

In the case of large numbers, it is convenient to express a partial permutation as a quotient of two factorials; however it is computed as a product of the declining factors. For example 100×99×98 = 100! divided by 97! This partial permutation is written \(100 \text{P}_3\) or as P(100, 3).

EXAMPLE How many ways are there to choose which four of ten pictures are to be placed in position?
This is a combination of four from ten because only the choice is important, not the order. Because any selection of four pictures has 4! arrangements, the number of combination multiplied by 4! must equal 5040. That is, the number of combinations of four from ten is \(10 \text{P}_4\) divided by 4! which equals 210 ways. It is written as \(10 \text{C}_4\) or as C(10,4).

EXAMPLE Given the set \{R, S, T\}, there are 3×2 = 6 permutations of 2 elements: RS, RT, SR, ST, TR, TS; but there are only 6/2 = 3 combinations of 2 elements: \{R, S\}, \{R, T\}, \{S, T\} because only the choices, not the order, matters.

The examples cited all involve choosing from a supply of distinguishable (distinct, different) objects. In some cases we wish to draw from an endless supply of identical objects of different types. Such cases are known as selections with replacement, as a given type might be chosen more than one time. A declining product is not appropriate in such cases.

EXAMPLE How many three letter arrangements [“words”] can be formed from an alphabet of 26 letters? Here the alphabet is 26 types, not 26 objects, so there are 26×26×26 which equals 17576 words.

Permutations without replacement from a limited supply of identical objects of different types is best handled with a tree diagram.
EXAMPLE The letters in LOOP can be used to form seven 2-letter words: LO, LP, OL, OO, OP, PL, PO

Doing combinations with replacement is a more complicated matter which will be explained in a later section.